Students' Name:____________________ Teachers' Code: Please circle NHM JPN MXF FEY KXM

Saint Ignatius' College, Riverview
Mathematics Assessment Task
2024

Year 12
Mathematics Advanced
Task 4 Trial HSC Exam
Date: 23rd August 2024

General Instructions:

- **Reading time : 10 mins**
- **Time Allowed: 3 hours**

- Write using blue or black pen only
- NESA approved calculators may be used
- Attempt all questions.
- A NESA Reference Sheet is provided.
- Questions 1 to 10 are all multiple-choice questions worth 1 mark each and are to be answered on the multiple-choice answer sheet provided.
- Questions 11 to 36 are each worth 90 marks and are to be answered on the examination paper.
- Each booklet and the multiple-choice answer sheet must have **your name** and **the initials of your class teacher** on the front cover.
- Marks may not be awarded for missing or carelessly arranged working.

Topics Examined:

All Preliminary and HSC Mathematics topics

SECTION 1 **Questions 1 - 10**	**10 Marks**
SECTION 2 **Question 11 - 36**	**90 Marks**
Total	**100 Marks**

Teacher:

Mr N Mushan	NHM
Ms F Yates	FEY
Mr J Newey	JPN
Mr P Collins	PPC
Ms K Mullan	KXM

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SECTION 1 (10 marks)

Attempt Questions 1 – 10

1. The domain of $y = \log_e(5-2x)$ is:

A. $x < -\frac{5}{2}$

B. $x > -\frac{5}{2}$

C. $x < \frac{5}{2}$

D. $x > \frac{5}{2}$

2. The new equation when $xy = 1$ is translated right by 3 units is:

A. $x = \frac{1}{y+3}$

B. $y = \frac{1}{x+3}$

C. $x = \frac{1}{y-3}$

D. $y = \frac{1}{x-3}$

3. At the NSW State Swimming Championships, the time in seconds for competitors from all age groups to finish the 50-metre freestyle is normally distributed with a mean of 27 seconds and a standard deviation of 1.5.

Calculate the percentage of 50-metre freestyle swim competitors who complete the lap in less than 24 seconds.

A. 34%

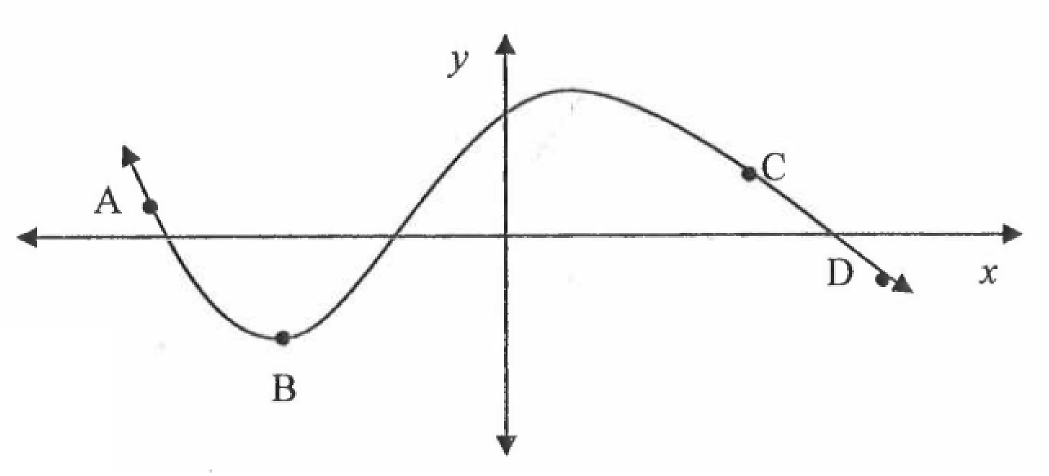
B. 13.5%

C. 2.5%

D. 0.15%

4. Which point on the following diagram relates to the following description.

$y > 0$, $\frac{dy}{dx} < 0$, $\frac{d^2y}{dx^2} < 0$



A. A

B. B

C. C

D. D

5. Evaluate $\int_{-1}^{1} \frac{1}{e^{2x}}\, dx$

A. $1 - e^2$

B. $e^2 - e^{-2}$

C. $\frac{e^3 - 1}{3}$

D. $\frac{e^2 - e^{-2}}{2}$

6. $\int_{-1}^{a}(2x+1)^3\,dx = 10$ where $a > 0$. The value of a is:

A. 1

B. 4

C. 6

D. 8

7. The curve $y = ax^2 + bx + 4$ has a stationary point at (3,-5). The values of *a* and *b* are:

A. $a = 3;\ b = -5$

B. $a = 1;\ b = -6$

C. $a = -1;\ b = 6$

D. $a = -3;\ b = 5$

8. The solutions to $\sec(x + \frac{\pi}{4}) = \sqrt{2}$ for $0 \le x \le 2\pi$ are:

$x =$

A. $\frac{\pi}{3}, \frac{5\pi}{3}$

B. $\frac{\pi}{2}, \frac{5\pi}{2}$

C. $0, \frac{3\pi}{2}, 2\pi$

D. $0, \frac{\pi}{4}, \frac{5\pi}{4}, 2\pi$

9. If $y = 4^{-x}$, then $\frac{dy}{dx} =$

A. 4^{-x}

B. $\frac{1}{x\log_e 4}$

C. -4^{-x-1}

D. $\frac{-\log_e 4}{4^x}$

10. If $\frac{d^2y}{dx^2} = 6x - 6$,

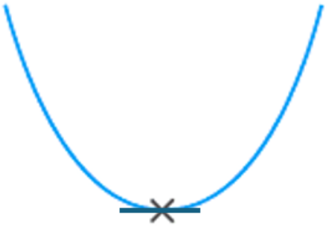
$\frac{dy}{dx} = 0$ when $x = 1$,

and when $x = 2$, $y = -4$

therefore, the nature of the stationary point at $x = 1$ is:

A.

B.



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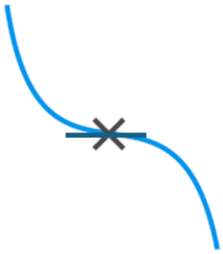

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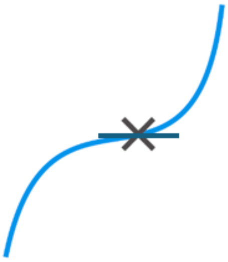

End of Section 1

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SECTION 2

Total Marks – 90
Attempt Questions 11-36
[Marks for each part are indicated on the page]
Allow about 2 hours and 45 minutes for this section

Question 11 **Marks**

A new car purchased for $27000 depreciates by 15% of its purchase price each year.

(a) What is the value of the car after 1 year? **1**

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(b) What is the value of the car after 3 years? (correct to 2 decimal places) **2**

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(c) When will the value of the car be half of the purchase price? **2**
(correct to 2 decimal places)

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Question 12

Find the 6th term of the geometric sequence if $T_2 = 3$ and $T_5 = \frac{81}{8}$ **2**

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Question 13

Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{1+\sin x}\,dx$. **2**

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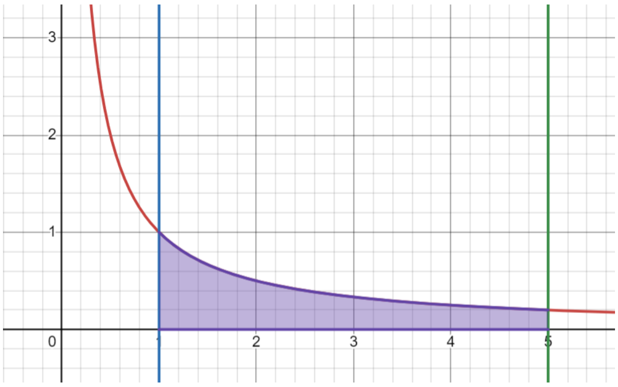
Question 14

A sketch of $y = \frac{1}{x}$ for the definite integral $\int_1^5 \frac{1}{x}\,dx$ is shown below.



Use the trapezoidal rule with 5 function values to estimate the area of the shaded region. **2**

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Question 15

Consider the Probability Density Function (PDF) given by

$$f(x) = \begin{cases} \frac{1}{x^2}, & x \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find $P(X \geq 3)$. **2**

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(b) Write down the formula for $P(A|B)$ **1**

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(c) Given $P(X \geq 2) = \frac{1}{2}$, hence, find $P(\, X \geq 3 \,|\, X \geq 2)$ **1**

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Question 16

For the curve $y = x^3 - 9x^2 + 24x - 15$

(a) Find the coordinates of the stationary points and determine their nature. **4**

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(b) Find the point of inflection. **2**

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(c) What is the global maximum and minimum of $y = x^3 - 9x^2 + 24x - 15$ for the domain $x \in R\ [0, 6]$? **2**

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Question 17

Differentiate $\frac{\sin x}{x^2}$. **2**

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Question 18

(a) Evaluate $\int_0^1 \frac{x-1}{x^2-2x+4}\,dx$. (Write your answer as an exact value) **3**

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(b) Evaluate $\int_{\ln 2}^{2\ln 3} \left(1-e^x\right)^2\,dx$. **2**

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Question 19

Simplify $\dfrac{\sin^2 x + \cos^2 x}{\cos^2 x}$. **1**

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Question 20

A particle undergoes straight line motion with velocity $v = \dfrac{6}{\sqrt{3t+4}}$

where t is time in seconds and distance is in metres.

(a) Find the particle’s position x at time t, if initially the particle is at the origin. **2**

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(b) Find the position of the particle 7 seconds later. **1**

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Question 21

The diagram shows the parabola $y = x^2 - 3x$ and a tangent drawn at P.
The equation of the tangent at P is $y = x - 4$ and it cuts the *y*-axis at Q.

(a) Show that the *x* co-ordinate of P is $x = 2$. **1**

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(b) Find the **co-ordinates** of Q. **1**

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(c) A normal is drawn from P and cuts the *y*-axis at R. **2**
Find the equation of the normal and the co-ordinates of R.

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(d) Find the area of triangle RPQ. **1**

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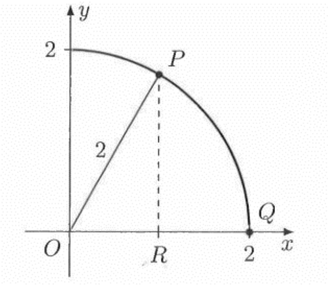
Question 22



The diagram shows a point P on a part of $y = \sqrt{4 - x^2}$.

The point P is vertically above R and Q has coordinates (2,0). The point R has coordinates (1,0).

(a) State in radians the size of angle POR. **1**

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(b) Calculate the area of sector OPQ. (Write your answer as an exact value) **2**

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(c) Calculate the area of triangle OPR. (Write your answer as an exact value) **1**

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(d) Hence find $\int_0^1 \sqrt{4 - x^2}\, dx$ (Write your answer as an exact value) **2**

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Question 23

A continuous random variable X has a probability density function $f(x)$ given by

$$f(x) = \begin{cases} 12x^2(1-x), & \text{for } 0 \leq x \leq 1 \\ 0, & \text{for all other values of } x \end{cases}.$$

(a) Find the mode of X. **2**

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(b) Find the cumulative distribution function for the given probability density function. **1**

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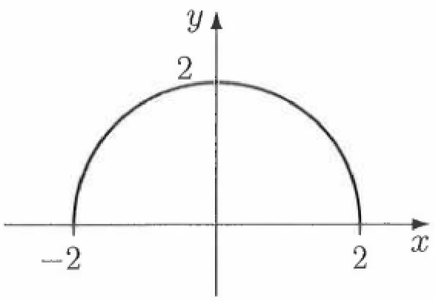
Question 24

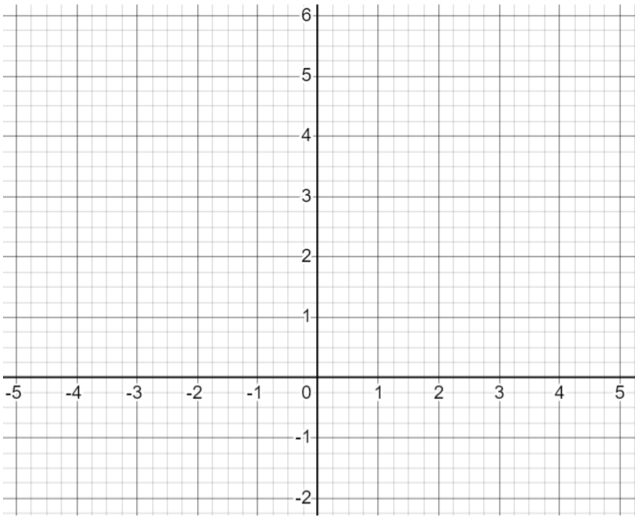
The diagram below shows the graph of $y = f(x)$



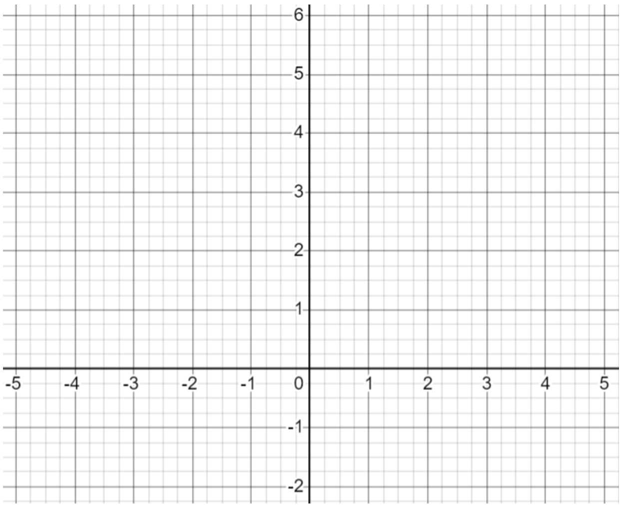
Sketch on the axes provided, the following **transformations** of the semi-circle,

showing the coordinates of the *x* and *y* intercepts.

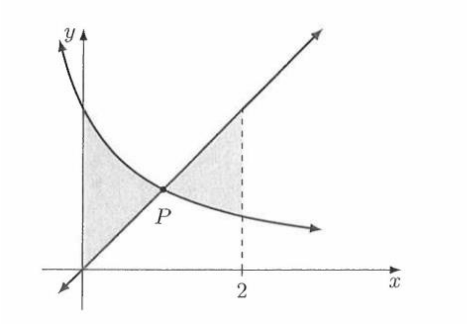
(a) $y = 2f(2x)$ **2**



(b) $y = -\frac{1}{2} f(\frac{x}{2})$ **2**



Question 25



The shaded region is bounded by the hyperbola $y=\frac{2}{x+1}$, and line $y=x$.

Point P is the intersection of the hyperbola and the line

(a) Show that the x-coordinate of P is 1. **1**

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(b) Find the exact area of the shaded regions. **3**

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Question 26

A mass is bouncing from the end of an elastic string and its height *h* (metres) above the ground at time *t* (seconds) is given by $h=1.6+0.4\cos 2\pi t$

(a) Between what heights is the mass bouncing between? **2**

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(b) What is the period between when the mass is at its highest point in consecutive bounces? **1**

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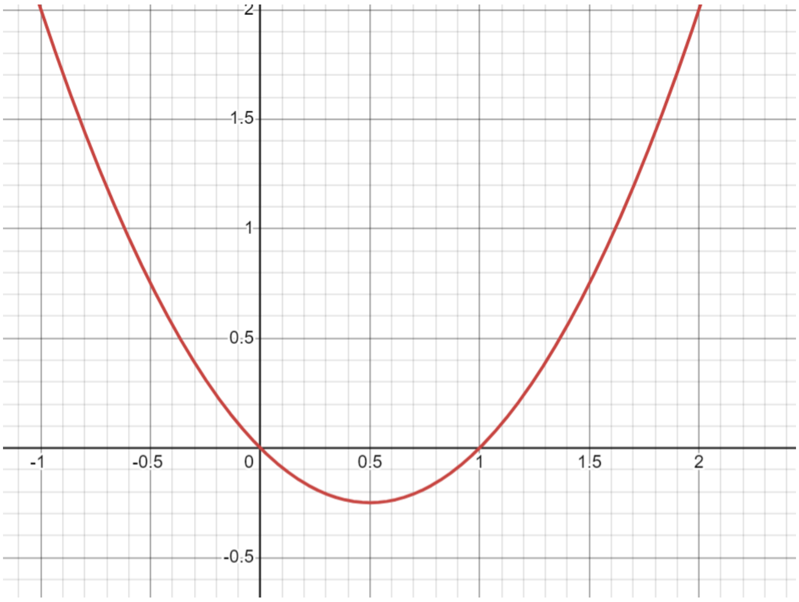
Question 27

(a) The graph of $y=f(x)$ is shown below.



On the graph above, sketch the curve, $y=f(2x-1)$, showing all important features. **2**

Question 28

A continuous random variable X has a probability density function $f(x)$ given by

$$f(x) = \begin{cases} \dfrac{1}{\sqrt{x}}, & 1 \leq x \leq k \\ 0, & \text{otherwise} \end{cases}$$

Find the value of k. **2**

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Question 29

Evaluate $\int_1^2 12x(x^2+3)^5\,dx$ **2**

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Question 30

Donielle completed two class tests this week.

Her results are shown in the table below (class mean and standard deviation shown for each subject).

Subject	**Donielle's Mark**	**Mean**	**Standard Deviation**
Mathematics	72	64	4
Chemistry	78	68	10

(a) In which test did Donielle perform better, relative to her class peers. **2**

(You must show working to support your answer)

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(b) Tina is in Donielle's class and sat the same class tests. Her z-score for Mathematics is 1.5. **1**

What mark did Tina record for Mathematics?

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Question 31

Find the equation of the curve if $\dfrac{d^2y}{dx^2} = -\dfrac{4}{(2x-1)^2}$. **4**

The curve passes through $(5, 2\ln 3)$ and the gradient of the tangent when $x = 1$ is 2.

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Question 32

Find the values of A and B so that $y = A\cos(2x) + B\sin(2x)$ satisfies the equation

$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 12y = \sin(2x)$ for any real values A and B. **2**

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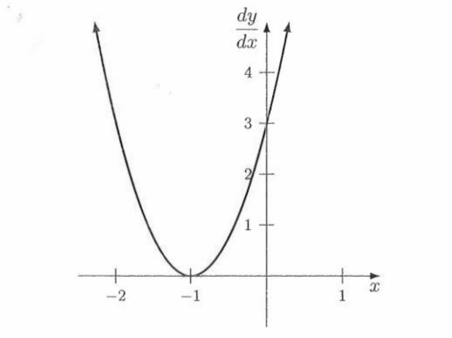
Question 33

The gradient function $\frac{dy}{dx} = 3(x+1)^2$ of a curve is shown below.



What is the nature of the stationary point at $x = -1$ on $y = f(x)$? **2**

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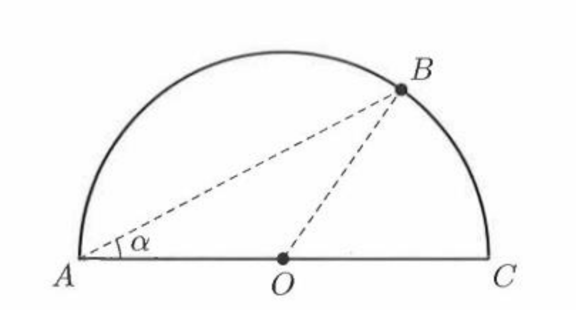
Question 34

A semi-circular garden with radius 1 km is surrounded by a path. Angela wishes to go from one end of the garden to another (that is move from A to C passing through B) in the maximum possible time to enjoy the ambience of the garden and get some gentle exercise. She decides to walk in a straight line from A to B at a pace of 2 kilometres per hour, and then jog arc BC at a pace of 4 kilometres per hour.



Let $\angle BAC = \alpha$

(a) Show that $\angle BOC = 2\alpha$, and hence show that Angela runs 2α kilometres. **2**

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(b) If line $AB = 2\cos\alpha$, show that the time taken for the total journey is $T = \cos\alpha + \frac{\alpha}{2}$. **1**

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(e) Find the value for α for which $\frac{dT}{d\alpha}=0$, and determine whether this gives the maximum or minimum value of T. **2**

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(f) Find how long (to the nearest minute) it takes Angela to complete her journey if she proceeds with the above path (that is move from A to C passing through B). **1**

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Question 35

If $x=6\cos(3t+\frac{3\pi}{4})$ and $\frac{d^2x}{dt^2}=-n^2x$. Find the value of *n*. **2**

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Question 36

Jean makes a *quarterly* deposit of $4000 into an account at the beginning of each quarter.
The account pays an interest rate of 6% per annum, compounded monthly.

(a) Show that Jean will have $16612.16 in the account at the end of the first year. **2**

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(b) Find the amount in the account at the end of the second year. **2**

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(c) If Jean wishes to have $250,000 in the account after 10 years,
how much should his quarterly deposit be? **2**

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End of Exam

Students' Name:____________________ Teachers' Code: Please circle NHM JPN MXF FEY KXM

SUGGESTED SOLUTIONS

Saint Ignatius' College, Riverview
Mathematics Assessment Task
2024

Year 12
Mathematics Advanced
Task 4 Trial HSC Exam
Date: 23rd August 2024

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- **Time Allowed: 3 hours**

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Topics Examined:

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SECTION 2 **Question 11 - 36**	**90 Marks**
Total	**100 Marks**

Teacher:

Mr N Mushan	NHM
Ms F Yates	FEY
Mr J Newey	JPN
Mr P Collins	PPC
Ms K Mullan	KXM

SECTION 1 (10 marks)

Attempt Questions 1 – 10

1. The domain of $y = \log_e(5-2x)$ is:

A. $x < -\frac{5}{2}$

B. $x > -\frac{5}{2}$

[✓ C.] $x < \frac{5}{2}$

D. $x > \frac{5}{2}$

$5 - 2x > 0$

$5 > 2x$

$\frac{5}{2} > x$

$x < \frac{5}{2}$

2. The new equation when $xy=1$ is translated right by 3 units is:

$y = \frac{1}{x} \Rightarrow y = \frac{1}{x-3}$

A. $x = \frac{1}{y+3}$

B. $y = \frac{1}{x+3}$

C. $x = \frac{1}{y-3}$

[✓ D.] $y = \frac{1}{x-3}$

3. At the NSW State Swimming Championships, the time in seconds for competitors from all age groups to finish the 50-metre freestyle is normally distributed with a mean of 27 seconds and a standard deviation of 1.5.

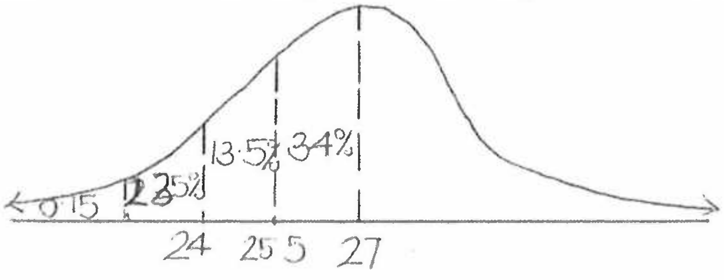
Calculate the percentage of 50-metre freestyle swim competitors who complete the lap in less than 24 seconds.

A. 34%

B. 13.5%

[✓ C.] 2.5%

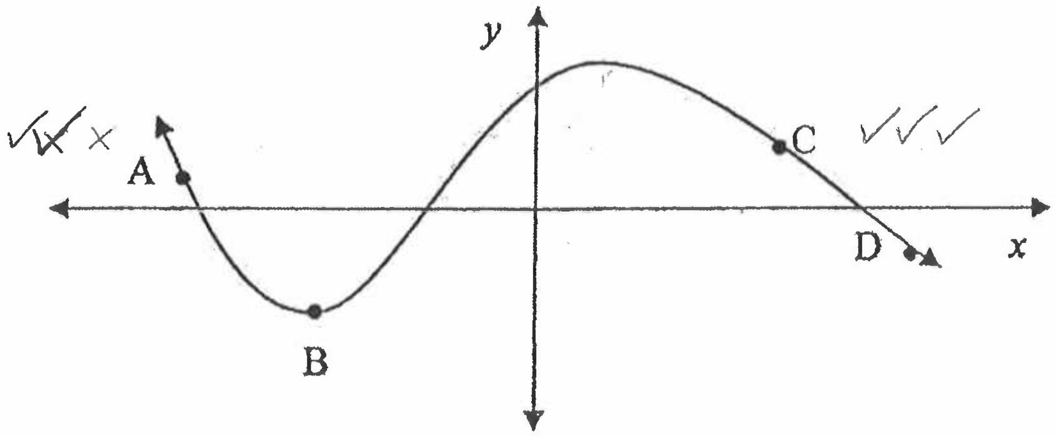
D. 0.15%



$$\begin{array}{r} 2.35 + \\ 0.15 \\ \hline 2.50\% \end{array}$$

4. Which point on the following diagram relates to the following description.

$y > 0,\ \frac{dy}{dx} < 0,\ \frac{d^2y}{dx^2} < 0$



A. A

B. B

[✓ C.] C

D. D

5. Evaluate $\int_{-1}^{1} \frac{1}{e^{2x}}\, dx$

A. $1-e^2$

B. e^2-e^{-2}

C. $\frac{e^3-1}{3}$

[✓ D)] $\frac{e^2-e^{-2}}{2}$

$\int e^{-2x}\,dx$

$\frac{1}{-2}\int -2e^{-2x}\,dx$

$-\frac{1}{2}\left[e^{-2x}\right]_{-1}^{1}$

$= -\frac{1}{2}\left[e^{-2}-e^{2}\right]$

$= \frac{1}{2}\left[e^{2}-e^{-2}\right]$

6. $\int_{-1}^{a}(2x+1)^3\,dx = 10$ where $a>0$. The value of a is:

$\frac{1}{8}\left[(2x+1)^4\right]_{-1}^{a} = 10$

$\left[(2x+1)^4\right]_{-1}^{a} = 80$

$(2a+1)^4 - (-1)^4 = 80$

$(2a+1)^4 = 81$

$2a+1 = \sqrt[4]{81} = 3$

$\therefore a = 1$.

[✓ A.] 1

B. 4

C. 6

D. 8

7. The curve $y = ax^2 + bx + 4$ has a stationary point at (3,-5). The values of *a* and *b* are:

$y = ax^2 + bx + C$ sub (3,5)

$\frac{dy}{dx} = 2ax + b$ sub $x = 3$ $\frac{dy}{dx} = 0$.

A. $a=3$; $b=-5$

[✓ B.] $a=1$; $b=-6$

C. $a=-1$; $b=6$

D. $a=-3$; $b=5$

$6a + b = 0$ .. ① $\times -3$

$9a + 3b = -9$ .. ②

$-18a - 3b = 0$ .. ③

$9a + 3b = -9$ .. ②

$-9a = -9$

$a = 1$.

$b = -6a$

$b = -6$.

8. The solutions to $\sec(x+\frac{\pi}{4}) = \sqrt{2}$ for $0 \le x \le 2\pi$ are:

$x =$

A. $\frac{\pi}{3}, \frac{5\pi}{3}$

B. $\frac{\pi}{2}, \frac{5\pi}{2}$

[✓ C.] $0, \frac{3\pi}{2}, 2\pi$

D. $0, \frac{\pi}{4}, \frac{5\pi}{4}, 2\pi$

$\frac{1}{\cos(x+\frac{\pi}{4})} = \frac{\sqrt{2}}{1}$

$\cos(x+\frac{\pi}{4}) = \frac{1}{\sqrt{2}}$

$\cos(x+\frac{\pi}{4}) = \cos(\frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4},\ )$

$x + \pi/4 = \pi/4,\ 7\pi/4,\ 9\pi/4,$.

$x = 0,\ \frac{6\pi}{4},\ 8\pi/4, \ldots$

$x = 0,\ 3\pi/2,\ 2\pi, \ldots$

9. If $y = 4^{-x}$, then $\frac{dy}{dx} =$

$y = a^{f(x)}$

$\frac{dy}{dx} = (\ln a)\, f'(x)\, a^{f(x)}$

$\frac{dy}{dx} = (\ln 4)\ -1\ 4^{-x}$

A. 4^{-x}

B. $\frac{1}{x\log_e 4}$

C. -4^{-x-1}

[✓ D.] $\frac{-\log_e 4}{4^x}$

10. If $\frac{d^2y}{dx^2} = 6x - 6$,

$\frac{dy}{dx} = 0$ when $x = 1$,

and when $x = 2$, $y = -4$

therefore, the nature of the stationary point at $x = 1$ is:

when $x=1$ $\frac{d^2y}{dx^2} = 6(1) - 6 = 0$

possible point of inflexion

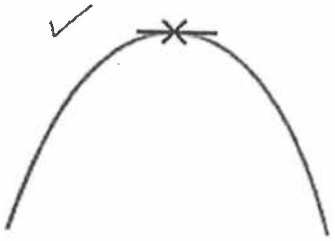
test:

x	0	1	2
$\frac{d^2y}{dx^2}$	-6	0	6

concave down

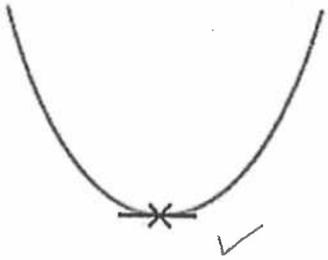
concave up

A.



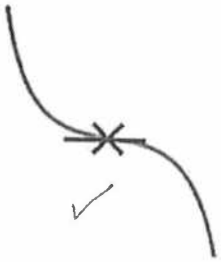
✓

B.



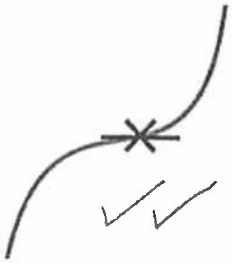
✓

C.



✓

[✓ D.]



✓✓

$y = x^3 - 3x^2 + 3x + D$

when $x=2$ $y=-4$

$D = -6$

$y = x^3 - 3x^2 + 3x - 6$

(sketch when $x=1$)

End of Section 1

ALTERNATIVE METHOD

$\frac{d^2y}{dx^2} = 6x - 6$

$\frac{dy}{dx} = 3x^2 - 6x + C$

when $x=1$ $C=3$

$\frac{dy}{dx} = 3x^2 - 6x + 3$

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SECTION 2

Total Marks – 90
Attempt Questions 11-36
[Marks for each part are indicated on the page]
Allow about 2 hours and 45 minutes for this section

Question 11 **Marks**

A new car purchased for $27000 depreciates by 15% of its purchase price each year.

(a) What is the value of the car after 1 year? 1

↘ depreciates 4050

$S = 27000(1-0.15)^1$

$= \$22950$ ✓

$27000 - 4050 = 22950$ ✓

(b) What is the value of the car after 3 years? (correct to 2 decimal places) 2

Common error

$S = 27000(1-0.15)^3$

$= \$16581.38$ ✓

Correct solution

~~✓✓~~ $S = 27000 - 3(4050)$

14850 ✓✓

(c) When will the value of the car be half of the purchase price? (correct to 2 decimal places) 2

Yo	Y1	YR2	YR3
27000	22950	18900	14850

Common error:

$13500 = 27000(1-0.15)^n$

$\frac{1}{2} = 0.85^n$ ~~✓✓~~

$\ln\frac{1}{2} = \ln 0.85^n$

$\ln\frac{1}{2} = n\ln 0.85$

$n = \frac{\ln\frac{1}{2}}{\ln 0.85}$

$n = 4.27$ (2 d.p) ✓✓ efpa.

Correct solution

$27000 - n(4050) = 13500$

$13500 = 4050n$

$n = 3.33$ ✓✓

Tn $27000 + (n-1)d = 13500$

$27000 + (n-1)-4050 = 13500$

$27000 - 4050n + 4050 = 13500$

$-4050n = -17550$

$n = 4.33$ (term number) ← Year is 1 less than term number

Question 12

Find the 6th term of the geometric sequence if $T_2 = 3$ and $T_5 = \frac{81}{8}$ **2**

$ar^4 = \frac{81}{8}$ ... ①　　　　$r = \frac{3}{2}$ } both ✓

$ar = 3$ ... ②　　　　$a = 2$

$r^3 = \frac{27}{8}$　　　　$T_6 = \frac{243}{16}$

Question 13

Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{1+\sin x}\, dx$. **2**

$= \left[\log_e(1+\sin x)\right]_{\pi/6}^{\pi/2}$ ✓

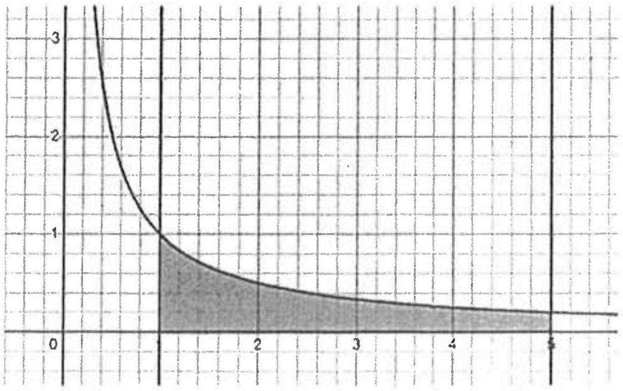
$= \left[\log_e(1+\sin\frac{\pi}{2}) - \log_e(1+\sin\frac{\pi}{6})\right]$

$= \log_e 2 - \log_e(\frac{3}{2})$ ✓

OR $\log_e(\frac{4}{3})$

Question 14

A sketch of $y = \frac{1}{x}$ for the definite integral $\int_1^5 \frac{1}{x}\, dx$ is shown below.



Use the trapezoidal rule with 5 function values to estimate the area of the shaded region. **2**

x	1	2	3	4	5
y	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$

✓

$TR = \int_1^5 \frac{1}{x}\, dx \doteqdot \frac{1}{2}\left[1 + 2(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}) + \frac{1}{5}\right]$

$= \frac{101}{60}$ $(\doteqdot 1{\cdot}683)$ ✓

Question 15

Consider the Probability Density Function (PDF) given by

$$f(x) = \begin{cases} \frac{1}{x^2}, & x \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find $P(X \geq 3)$. **2**

$\int_3^\infty \frac{1}{x^2}\,dx$

$= \int_3^\infty x^{-2}\,dx$

$= \left[\frac{-1}{x}\right]_3^\infty$ ✓

$= \frac{-1}{\infty} + \frac{1}{3}$

$= 0 + 1/3$

$= 1/3$ ✓

OR $\int_1^3 \frac{1}{x^2}\,dx$

$\int_1^3 x^{-2}\,dx$

$= \left[\frac{-1}{x}\right]_1^3$

$= -\frac{1}{3} - -1$

$= 2/3$ ✓

$P(X \geqslant 3) = 1 - 2/3 = 1/3$ ✓

(b) Write down the formula for $P(A|B)$ **1**

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ ✓

(c) Given $P(X \geq 2) = \frac{1}{2}$, hence, find $P(X \geq 3 \mid X \geq 2)$ **1**

$P(X \geqslant 3 | X \geqslant 2) = \frac{P(X \geqslant 3 \text{ and } X \geqslant 2)}{P(X \geqslant 2)}$ is $\frac{P(X \geqslant 3)}{P(X \geqslant 2)} = \frac{1/3}{\frac{1}{2}} = \frac{2}{3}$ ✓

Note $P(X \geqslant 3 \text{ and } X \geqslant 2) \neq P(X \geqslant 3) \times P(X \geqslant 2)$

unless independent!

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Question 16

For the curve $y = x^3 - 9x^2 + 24x - 15$

(a) Find the coordinates of the stationary points and determine their nature. 4

S.P occur when $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 3x^2 - 18x + 24$

$\therefore 3(x^2 - 6x + 8) = 0$

$3(x-2)(x-4) = 0$

$x = 2, 4$

when $x = 2$ $y = 5$ $(2,5)$

when $x = 4$ $y = 1$ $(4,1)$

Nature of SP

$\frac{d^2y}{dx^2} = 6x - 18$

when $x = 2$ $\frac{d^2y}{dx^2} = -6$ concave down ✓

✓ $\therefore (2,5)$ is a local max T.P

when $x = 4$ $\frac{d^2y}{dx^2} = 6$ concave up

$(4,1)$ is a local min T.P ✓ ✓

Alternate values est

1	x	0	2	3	4	5
9	y'	24	0	-3	0	9

← With these tests you MUST SUPPLY VALUES AND make it clear which values you have in your tables x, y', y'', $\frac{dx}{dt}$, ...

(b) Find the point of inflection. 2

Possible Point of inflexion $\frac{d^2y}{dx^2} = 0$

$6x - 18 = 0$ ✓

$x = 3$ $(3,3)$

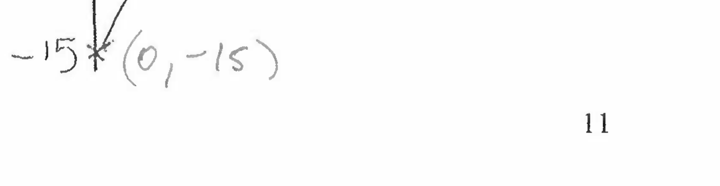
MUST SEE TEST →

test x	2	3	4
$\frac{d^2y}{dx^2}$	-6	0	6

✓

$\therefore$ change in concavity ∩ ∪ $\therefore (3,3)$ is a point of inflexion

(c) What is the global maximum and minimum of $y = x^3 - 9x^2 + 24x - 15$ for the domain $x \in R\ [0,6]$? 2



Global Max. $(6,21)$ ✓

Global Min $(0,-15)$ ✓

Question 17

Differentiate $\frac{\sin x}{x^2}$. $\frac{\text{uside} - \text{outside}}{v^2}$ **2**

marks awarded at this stage

$$= \frac{x^2\cos x - 2x\sin x}{x^4}$$ ✓ ✓

$u = \sin x$

$u' = \cos x$

$v = x^2$

$v' = 2x$

$$= \frac{x\cos x - 2\sin x}{x^3}$$

do recommand the simplification

Question 18

(a) Evaluate $\int_0^1 \frac{x-1}{x^2-2x+4}\,dx$. (Write your answer as an exact value) **3**

$(-0.1438 \quad 4\text{ sig fig})$

$$\frac{1}{2}\int_0^1 \frac{2x-2}{x^2-2x+4}\,dx$$ ✓

$$= \frac{1}{2}\left[\ln(x^2-2x+4)\right]_0^1$$ ✓

$$= \frac{1}{2}\left[\ln 3 - \ln 4\right]$$ ✓

OR

* equivalents $\frac{1}{2}(\ln 3) - \ln 2$ OR $\frac{1}{2}\ln\left(\frac{3}{4}\right)$ OR $\ln\left(\frac{\sqrt{3}}{2}\right)$

(b) Evaluate $\int_{\ln 2}^{2\ln 3} \left(1-e^x\right)^2\,dx$. **2**

$$\int_{\ln 2}^{\ln 9} 1 - 2e^x + e^{2x}\,dx$$

$$= \left[x - 2e^x + \frac{1}{2}e^{2x}\right]_{\ln 2}^{\ln 9}$$ ✓

$$= \left[\left\{\ln 9 - 2(9) + \frac{1}{2}(81)\right\} - \left\{\ln 2 - 2(2) + \frac{1}{2}(4)\right\}\right]$$

$$= \ln 9 - \ln 2 - 18 + \frac{81}{2} + 4 - 2$$

$$= \ln\left(\frac{9}{2}\right) + \frac{49}{2}$$ ✓

$[\doteqdot 26.004]$

Question 19

Simplify $\frac{\sin^2 x+\cos^2 x}{\cos^2 x}$. **1**

$= \frac{1}{\cos^2 x} = \sec^2 x$ ✓

$= \tan^2 x + 1$

Question 20

A particle undergoes straight line motion with velocity $v=\frac{6}{\sqrt{3t+4}}$

where t is time in seconds and distance is in metres.

(a) Find the particle's position x at time t, if initially the particle is at the origin. **2**

$\frac{dx}{dt} = 6(3t+4)^{-\frac{1}{2}}$

$x = 6\int (3t+4)^{-\frac{1}{2}} dt$

$x = 6\left[\frac{(3t+4)^{\frac{1}{2}}}{3\times\frac{1}{2}}\right] + C$

✓

$x = 4\sqrt{3t+4} + C$

$\therefore x = 4\sqrt{3t+4} - 8$

when $t=0$ $x=0$

$0 = 4\sqrt{4} + C$

$C = -8$. ✓

(b) Find the position of the particle 7 seconds later. **1**

when $t=7$

$x = 4\sqrt{3(7)+4} - 8$

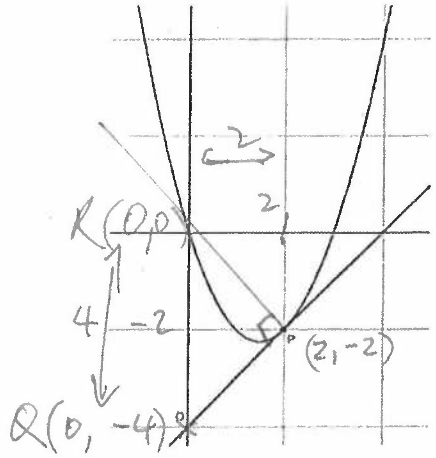
$= 4\sqrt{25} - 8$ ✓

$= 12m$

(to the right of the origin)

Question 21

The diagram shows the parabola $y = x^2 - 3x$ and a tangent drawn at P.
The equation of the tangent at P is $y = x - 4$ and it cuts the y-axis at Q.



(a) Show that the x co-ordinate of P is $x = 2$. **1**

$x^2 - 3x = x - 4$

$x^2 - 4x + 4 = 0$ ✓

$(x-2)^2 = 0$

$\therefore x = 2$

(b) Find the **co-ordinates** of Q. **1**

when $x = 0$ $y = 0 - 4 = -4$

Co-ordinates ie both x and y

$Q(0, -4)$ ✓

This page generally well done

(c) A normal is drawn from P and cuts the y-axis at R.
Find the equation of the normal and the co-ordinates of R. **2**

$y = x^2 - 3x$ $m_N = -1$

$m_T = \frac{dy}{dx} = 2x - 3$ $P(2, -2)$

when $x = 2$ $y + 2 = -1(x - 2)$

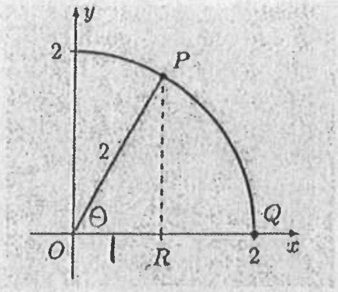
$m_T = 1$ $y = -x$ ✓ $R(0,0)$ ✓

(d) Find the area of triangle RPQ. **1**

$A = \frac{1}{2} \times 4 \times 2$

$= 4$ units^2 ✓

Question 22



The diagram shows a point P on a part of $y=\sqrt{4-x^2}$.

The point P is vertically above R and Q has coordinates (2,0). The point R has coordinates (1,0).

(a) State in radians the size of angle POR. **1**

$\cos\theta = \frac{1}{2}$ $\theta = \frac{\pi}{3}$ ✓ Correct answer only.

(b) Calculate the area of sector OPQ. (Write your answer as an exact value) **2**

$A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}(2)^2\left(\frac{\pi}{3}\right)$ ✓

$= \frac{2\pi}{3}$ units^2 ✓

many did not remember Area of sector formula.

(c) Calculate the area of triangle OPR. (Write your answer as an exact value) **1**

$A = \frac{1}{2}bh$

$A = \frac{1}{2} \times 1 \times \sqrt{3}$

$= \frac{\sqrt{3}}{2}$ ✓

$A = \frac{1}{2}ab\sin C$

OR $\frac{1}{2}(2)(1)\sin\left(\frac{\pi}{3}\right)$

$= \frac{\sqrt{3}}{2}$

(d) Hence find $\int_0^1 \sqrt{4-x^2}\,dx$ (Write your answer as an exact value) **2**

= sector OP(y-axis) + triangle

$= \frac{1}{2}(2)^2\left(\frac{\pi}{6}\right)$ ✓

$= \frac{\pi}{3} + \frac{\sqrt{3}}{2}$ ✓

$= \left(\frac{2\pi + 3\sqrt{3}}{6}\right)$

OR quarter semi-circle −(sector−trian

$= \pi - \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$= \pi - \frac{2\pi}{3} + \frac{\sqrt{3}}{2}$

$= \frac{\pi}{3} + \frac{\sqrt{3}}{2}$.

Question 23

A continuous random variable X has a probability density function $f(x)$ given by

$$f(x) = \begin{cases} 12x^2(1-x), & \text{for } 0 \le x \le 1 \\ 0, & \text{for all other values of } x \end{cases}.$$

(a) Find the mode of X. **2**

For mode let $f'(x) = 0$

$f(x) = 12x^2 - 12x^3$

$f'(x) = 24x - 36x^2$ ✓

$0 = 12x(2 - 3x)$

$x = 0, \frac{2}{3}$

MIN. MAX.

$f''(x) = 24 - 72x$

when $x = \frac{2}{3}$

$f''(x) = 24 - 72 \times \frac{2}{3} = -24$

Since $f''(x) < 0$ ∴ maximum occu

when $x = \frac{2}{3}$

∴ mode is $^2/_3$ only ✓

(b) Find the cumulative distribution function for the given probability density function. **1**

$F(x) = \int_0^x (12x^2 - 12x^3)\,dx$

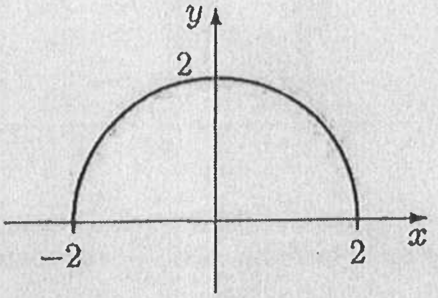
$= \left[4x^3 - 3x^4\right]_0^x$ ✓

$= (4x^3 - 3x^4) - (0 - 0)$

CDF $= F(x) = \underline{4x^3 - 3x^4}$ (OR ✓)

Question 24

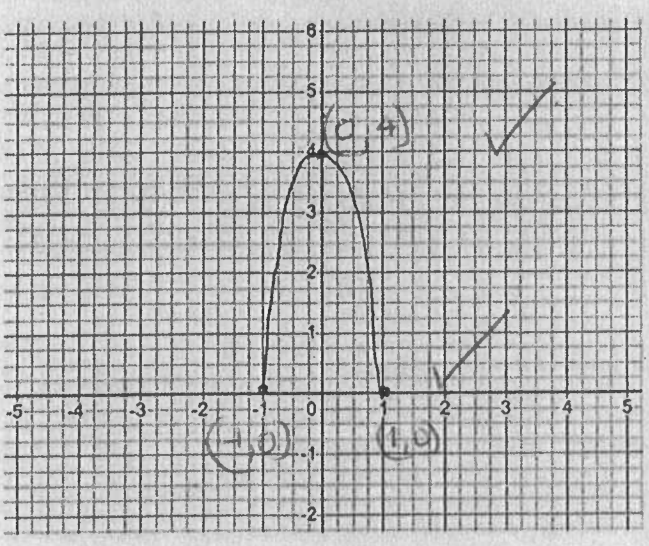
The diagram below shows the graph of $y = f(x)$



Sketch on the axes provided, the following **transformations** of the semi-circle,

showing the coordinates of the x and y intercepts.

(a) $y = 2f(2x)$ **2**



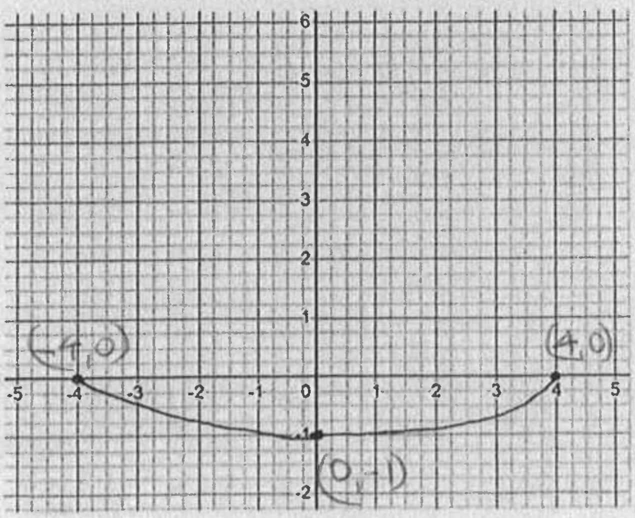
✓

✓

horizontal dilation
scale factor $\frac{1}{2}$

vertical dilation
scale factor 2

(b) $y = -\frac{1}{2}f(\frac{x}{2})$ **2**



✓

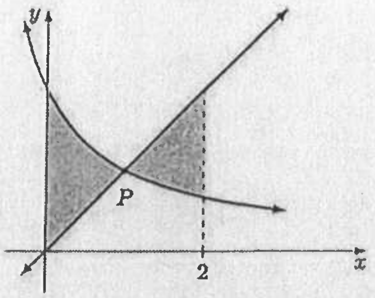
✓

horizontal dilation
scale factor 2

vertical dilation
scale factor $\frac{1}{2}$

reflection in the x-axis

Question 25



The shaded region is bounded by the hyperbola $y = \frac{2}{x+1}$, and line $y = x$.

Point P is the intersection of the hyperbola and the line

(a) Show that the x-coordinate of P is 1. **1**

$\frac{x}{1} = \frac{2}{x+1}$ $\qquad$ $x = -2, 1$

$x^2 + x = 2$ $\qquad$ in the 1st quadrant $\qquad$ 1

$x^2 + x - 2 = 0$ ~~(✓)~~ $\qquad$ so $x = 1$ only

$(x+2)(x-1) = 0$ ✓

(b) Find the exact area of the shaded regions. **3**

Right shaded Area $= \int_1^2 x - \frac{2}{x+1} \, dx$

$= \left[\frac{x^2}{2} - 2\ln|x+1| \right]_1^2$ ✓

$= \left[(2 - 2\ln 3) - (\tfrac{1}{2} - 2\ln 2) \right]$

$= \frac{3}{2} - 2\ln 3 + 2\ln 2$ $\quad$ $\left[\text{or } \frac{3}{2} - 2\ln\left(\frac{3}{2}\right)\right]$

Left shaded Area $= \int_0^1 \frac{2}{x+1} - x \, dx$

$= \left[2\ln|x+1| - \frac{x^2}{2} \right]_0^1$ ✓

$= \left[(2\ln 2 - \tfrac{1}{2}) - (2\ln 1 - 0) \right]$

$= 2\ln 2 - \frac{1}{2}$

total area $= 1 + 4\ln 2 - 2\ln 3$

OR

$1 + 2\left[\ln 2 - \ln\left(\frac{3}{2}\right)\right] = 1 + 2\ln\left(\frac{4}{3}\right)$

OR $\quad 1 + \ln\left(\frac{16}{9}\right)$ $\quad (\doteqdot 1{\cdot}57)$ ✓

Also $2(\ln 4 - \ln 3) + 1$

or $2\ln\frac{4}{3} + 1$

or $\ln\frac{16}{9} + 1$

Students' Name:____________________Teachers' Code: Please circle NHM JPN FEY PPC KXM

Question 26

A mass is bouncing from the end of an elastic string and its height h (metres) above the ground at time t (seconds) is given by $h = 1.6 + 0.4\cos 2\pi t$

(a) Between what heights is the mass bouncing between? 2

$0.4(1) + 1.6 = 2$ ✓ well done

$0.4(-1) + 1.6 = 1.2$ ✓

(b) What is the period between when the mass is at its highest point in consecutive bounces? 1

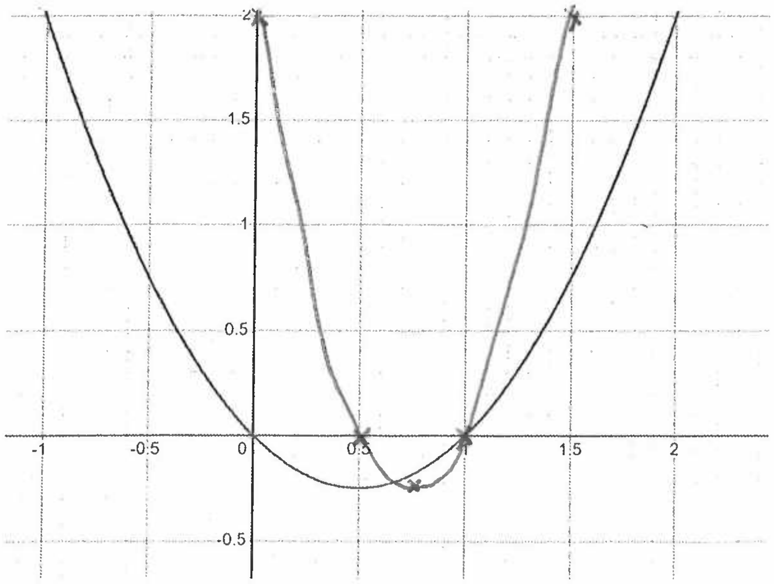
$\text{period} = \frac{2\pi}{a}$ well done.

$= \frac{2\pi}{2\pi}$ period $= 1$ ✓

$= 1$

Question 27 Not well answered!

(a) The graph of $y = f(x)$ is shown below.



Recommended method

$y = f(2(x - \frac{1}{2}))$ ←

$a = 2$ horizontal dilation SF $\frac{1}{2}$. ✓

then

$b = -\frac{1}{2}$ vertical translation $\frac{1}{2}$ to right. ✓

$(0,0) \to (\frac{1}{2},0)$
$(1,0) \to (1,0)$
$(-1,2) \to (0,2)$
Pick clear points and transform them.

OR

1 to right then horizontal dilation SF $\frac{1}{2}$

On the graph above, sketch the curve, $y = f(2x-1)$, showing all important features. 2

Question 28

A continuous random variable X has a probability density function $f(x)$ given by

$$f(x) = \begin{cases} \dfrac{1}{\sqrt{x}}, & 1 \le x \le k \\ 0, & \text{otherwise} \end{cases}$$

Find the value of k. **2**

$\int_1^k x^{-1/2}\,dx = 1$

careless istakes here ←

$\left[2x^{1/2}\right]_1^k = 1$

$\left[2\sqrt{x}\right]_1^k = 1$

$2\sqrt{k} - 2 = 1$

$2\sqrt{k} = 3$

$\sqrt{k} = \frac{3}{2}$

$k = \frac{9}{4}$

Careless mistakes here

± not needed when squaring.

Question 29

Evaluate $\int_1^2 12x(x^2+3)^5\,dx$ **2**

$= 6\int_1^2 2x(x^2+3)^5\,dx$

$= \left[\dfrac{\cancel{6}(x^2+3)^6}{\cancel{6}}\right]_1^2$

Several students were unable to do this step. Reverse Chain rule!

From reference sheet:

$\int f'(x)\,[f(x)]^n\,dx = \frac{1}{n+1}[f(x)]^{n+1} + c$

$= \left[(x^2+3)^6\right]_1^2$ ✓

$= 7^6 - 4^6$

$= 113553.$ ✓

Question 30

Donielle completed two class tests this week.

Her results are shown in the table below (class mean and standard deviation shown for each subject).

Subject	**Donielle's Mark**	**Mean**	**Standard Deviation**
Mathematics	72	64	4
Chemistry	78	68	10

(a) In which test did Donielle perform better, relative to her class peers. **2**

(You must show working to support your answer)

Maths $z = \frac{72-64}{4} = 2$

Chemistry $z = \frac{78-68}{10} = 1$ ✓

It is best to calculate both z-scores first and then compare.

Maths is better (higher z-score) ✓

(b) Tina is in Donielle's class and sat the same class tests. Her *z*-score for Mathematics is 1.5. **1**

What mark did Tina record for Mathematics?

$\frac{x-64}{4} = 1.5$

$x - 64 = 6$

Well done.

$x = 70$ ✓

Question 31

Some didn't spot this

Find the equation of the curve if $\frac{d^2y}{dx^2} = -\frac{4}{(2x-1)^2}$. **4**

The curve passes through $(5, 2\ln 3)$ and the gradient of the tangent when $x = 1$ is 2.

$y'' = -4(2x-1)^{-2}$ → You need to move denominator up first.

$y' = \frac{-4(2x-1)^{-1}}{-1 \times 2} + c$ → Careless mistakes here.

$= \frac{2}{2x-1} + c$ ✓ Many struggled to get to this stage. Integration needs work!

When $x = 1$ $y' = 2$.

$2 = \frac{2}{1} + c$

$c = 0$ ✓

$y' = \frac{2}{2x-1}$ Several struggled with integrating here.

$y = \ln|2x-1| + c$ ✓

when $x = 5$ $y = 2\ln 3$

$2\ln 3 = \ln 9 + c$

$2\ln 3 - \ln 9 = c$

$2\ln 3 - 2\ln 3 = c$

$c = 0$ $y = \ln|2x-1|$ ✓ use this notation as per reference sheet.

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Question 32

Find the values of A and B so that $y = A\cos(2x) + B\sin(2x)$ satisfies the equation

$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 12y = \sin(2x)$ for any real values A and B. **2**

$y = A\cos 2x + B\sin 2x$

$\left.\begin{aligned} y' &= -2A\sin 2x + 2B\cos 2x \\ y'' &= -4A\cos 2x - 4B\sin 2x \end{aligned}\right\}$ 1 mk

$y'' - 4y' - 12y = \sin 2x$

$(-4A\cos 2x - 4B\sin 2x) - 4(-2A\sin 2x + 2B\cos 2x) - 12(A\cos 2x + B\sin 2x)$

$= -4A\cos 2x - 4B\sin 2x + 8A\sin 2x - 8B\cos 2x - 12A\cos 2x - 12B\sin 2x$

$= (-4A - 8B - 12A)\cos 2x + (-4B + 8A - 12B)\sin 2x$

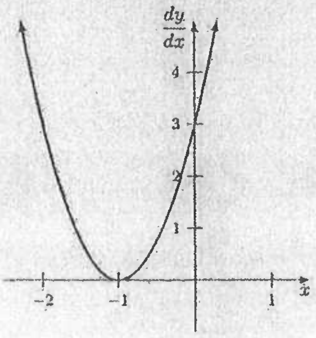
$= (-16A - 8B)\cos 2x + (8A - 16B)\sin 2x$

$\therefore \left.\begin{aligned} -16A - 8B &= 0 \\ 8A - 16B &= 1 \end{aligned}\right\} \quad \left.\begin{aligned} B &= -\frac{1}{20} \\ A &= \frac{1}{40} \end{aligned}\right\}$ 1mk

VERY FEW GOT HERE

Question 33

The gradient function $\frac{dy}{dx} = 3(x+1)^2$ of a curve is shown below.



What is the nature of the stationary point at $x = -1$ on $y = f(x)$? **2**

1mk awarded for POI

$y' = 3(x+1)^2 \quad x = -1 \quad y' = 0$

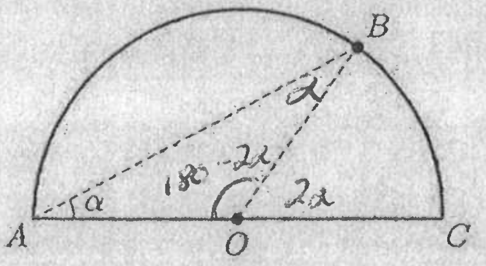
$y'' = 6(x+1) \quad x = -1 \quad y'' = 0$

Note change in concavity

$\therefore$ HPOI

Question 34

A semi-circular garden with radius 1 km is surrounded by a path. Angela wishes to go from one end of the garden to another (that is move from A to C passing through B) in the maximum possible time to enjoy the ambience of the garden and get some gentle exercise. She decides to walk in a straight line from A to B at a pace of 2 kilometres per hour, and then jog arc BC at a pace of 4 kilometres per hour.



Let $\angle BAC = \alpha$

(a) Show that $\angle BOC = 2\alpha$, and hence show that Angela runs 2α kilometres. **2**

$\angle ABO = \alpha$ 1mk $BC = r\theta$

$\therefore \angle BOC = 2\alpha$ $= 1.2\alpha$ } 1mk

$= 2\alpha$

(b) If line $AB = 2\cos\alpha$, show that the time taken for the total journey is $T = \cos\alpha + \frac{\alpha}{2}$. **1**

$T = \frac{D}{S}$

$= \frac{2\cos\alpha}{2} + \frac{2\alpha}{4}$ 1mk

$= \cos\alpha + \frac{\alpha}{2}$

(e) Find the value for α for which $\frac{dT}{d\alpha}=0$, and determine whether this gives the maximum or minimum value of T. **2**

$T = \cos\alpha + \frac{\alpha}{2}$

$\frac{dT}{d\alpha} = -\sin\alpha + \frac{1}{2}$

Let $\frac{dT}{d\alpha} = 0$

$\sin\alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{6}$ only 1mk

$\frac{d^2T^2}{d\alpha^2} = -\cos\alpha$

$\alpha = \frac{\pi}{6}$ $\frac{d^2T}{d\alpha^2} = -\frac{\sqrt{3}}{2}$

1mk for test.

$\therefore \alpha = \frac{\pi}{6}$

gives maximum

(f) Find how long (to the nearest minute) it takes Angela to complete her journey if she proceeds with the above path (that is move from A to C passing through B). **1**

$T = \cos\frac{\pi}{6} + \frac{\pi}{12}$

$= 1.12$ hrs

$= 68$ min 1mk

TOO MANY DID NOT USE RADIANS

Question 35

If $x=6\cos(3t+\frac{3\pi}{4})$ and $\frac{d^2x}{dt^2}=-n^2x$. Find the value of n. **2**

$\dot{x} = -18\sin(3t + \frac{3\pi}{4})$

$\ddot{x} = -54\cos(3t + \frac{3\pi}{4})$ 1mk

$= -9[6\cos(3t + \frac{3\pi}{4})]$

$= -9x$

$\therefore -9 = n^2$

$n = 3$ 1mk

Question 36

Jean makes a *quarterly* deposit of \$4000 into an account at the beginning of each quarter. The account pays an interest rate of 6% per annum, compounded monthly. She wishes to save up \$20 000 for a new car.

(a) Show that Jean will have \$16612.16 in the account at the end of the first year. **2**

START OF QUARTER

$$\left.\begin{array}{l} 1 : 4000(1.005)^{12} \\ 2 : 4000(1.005)^{9} \\ \vdots \quad \vdots \\ 4 : 4000(1.005)^{3} \end{array}\right\} 1\text{mk}$$

1mk

$$\text{TOTAL} = 4000(1.005^3 + \ldots + 1.005^{12})$$

$$= 16612.16$$

(b) Find the amount in the account at the end of the second year. **2**

2ND YEAR

$$T = 4000\overbrace{(1.005^3 + \ldots + 1.005^{24})}^{1\text{mk}}$$

$$= 34248.93 \quad 1\text{mk}$$

(c) If Jean wishes to have \$250,000 in the account after 10 years, how much should his quarterly deposit be? **2**

$$250000 = X\left[1.005^3 + 1.005^6 + \ldots + 1.005^{120}\right]$$

$$= X\left[\frac{1.005^3(1.005^{120} - 1)}{1.005^3 - 1}\right] \quad 1\text{mk}$$

$$= 4531.15$$

1mk

End of Exam